

## On a Medieval Solution to the Liar Paradox

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In this paper, I examine a solution to the Liar paradox found in the work of Ockham, Burley, and Pseudo-Sherwood. I reject the accounts of this solution offered by modern commentators. I argue that this medieval line suggests a non-hierarchical solution to the Liar, according to which 'true' is analysed as an indexical term, and paradox is avoided by minimal restrictions on tokens of 'true'. In certain respects, this solution resembles the recent approaches of Charles Parsons and Tyler Burge; in other respects, it is related to a suggestion of Gödel. But, as a whole, it suggests an original solution to the Liar paradox, quite unlike any current proposals.

### I. Introduction

Suppose I say:

This very sentence is not true.

If the sentence is true, then what I say is the case. But what I say is that the sentence is not true; and so the sentence is not true. But that is just what I say, and so the sentence is true after all.

This is one formulation of the Liar paradox.<sup>1</sup> According to Diogenes Laertius, the Liar was authored by Eubulides, in the 4th century B.C.<sup>2</sup> Little is known of ancient solutions. Aristotle appears to treat it as a fallacy *secundum quid et simpliciter*;<sup>3</sup> Chrysippus seems to have held that the Liar sentence has no meaning at all;<sup>4</sup> other treatments are alluded to in three entries of Diogenes's catalogue of the writings of Chrysippus.<sup>5</sup>

In contrast, the medieval *insolubilia* literature is a very rich source of attempts to solve the Liar.<sup>6</sup> Some of these resolutions are primarily influenced by Aristotle, and

- <sup>1</sup> In the terminology of the recent literature on the Liar, this is the Strengthened Liar. For more on the Strengthened Liar, see section 4 below. The 'Ordinary' Liar is in terms of falsity, rather than non-truth; it is generated, for example, by the sentence 'This sentence is false'.
- <sup>2</sup> Diogenes Laertius, *Lives of eminent philosophers* (trans. R. D. Hicks: Cambridge, Mass., Harvard University Press, 1930), Vol. 2, 108.
- <sup>3</sup> Aristotle, *Sophistical refutations* (trans. E. S. Forster: Cambridge, Mass., Harvard University Press, 1955), 180a27-67.
- <sup>4</sup> See I. M. Bochenski, *A history of formal logic* (Notre Dame, University of Notre Dame Press, 1961), 133.
- <sup>5</sup> Diogenes Laertius, *Lives* (footnote 2), vol. 3, 196-197.
- <sup>6</sup> An indispensable survey of medieval approaches to the Liar is provided by Paul Spade's *The medieval liar: a catalogue of the insolubilia-literature* (Toronto, 1975); henceforth in references, *A catalogue*.

attempt to force *insolubilia* into the mould of one or other of several fallacies laid out by Aristotle in *Sophistical refutations*.<sup>7</sup> Other solutions are distinctively medieval.<sup>8</sup> Still others anticipate, to varying degrees, modern approaches.<sup>9</sup> There are clear anticipations of the following modern views: the Liar sentence is not grammatically well-formed;<sup>10</sup> the Liar sentence, though well-formed, does not express a proposition;<sup>11</sup> the Liar sentence is neither true nor false, but suffers a truth-value gap, or else receives a third truth value;<sup>12</sup> and, finally, restrictions are to be placed on self-

Spade's paper, 'Insolubilia', in *The Cambridge history of later medieval philosophy* (ed. Norman Kretzmann, Anthony Kenny and Jan Pinborg: Cambridge, University Press, 1982), 246-253, is a very useful summary of medieval work on the Liar. Although I take issue with Spade at various points in this paper, it should be clear that I am indebted to his extensive work on the medieval Liar.

Paul of Venice's *Logica Magna* II, 15, provides another useful survey. Paul discusses and rejects fourteen previous opinions, before turning to his own. In the brief survey presented below, all quotes from Paul's treatise are taken from an unpublished manuscript by Marilyn Adams; the translation is of the Venice 1499 edition. A terminological note: 'First opinion' is to be understood as 'The first opinion discussed by Paul of Venice', and so on.

7 Medieval authors invoked the following Aristotelian fallacies: the fallacy *figuræ dictionis* (First opinion, f. 192RB); the fallacy *non causam ut causam* (Second opinion, f. 192RB); the fallacy of equivocation (Seventh opinion, f. 192VA); the fallacy *accidentis* (Fourteenth opinion, f. 194RAB); and the fallacy *secundam quæ et simpliciter* (Tenth opinion, f. 192VB).

8 One resolution labelled *transcensus* claims that the insoluble makes reference not to itself but to an utterance immediately preceding it (Third opinion, f. 192VA). Another builds on Aristotle's fallacy of equivocation, and distinguishes an utterance 'in process' from a 'conceived' utterance (Seventh opinion, f. 192VA). According to Paul's brief description, a further view holds that '... an insoluble is true or false, but is not true and not false' (Ninth opinion, f. 192VB). The views of Bradwardine, Swineshead, Heytesbury, and Peter of Ailly were original and influential. The Tenth opinion (f. 192VB), may be Bradwardine's (see Spade, *A catalogue*, 82); the Eleventh opinion (f. 192VB) is also close to Bradwardine's. Paul's own solution (Fifteenth opinion, f. 194Bff), is an elaboration of Swineshead's, as Spade points out (*A catalogue*, 83). The Twelfth opinion (f. 192VB-193RB) is Heytesbury's. The Thirteenth opinion (f. 193RB) is Peter of Ailly's.

9 This tripartite division is not altogether sharp. Some authors were influenced by Aristotle, but offered original suggestions. And some anticipations of modern approaches were placed within an Aristotelian framework.

10 According to the *casantes*, he who utters an insoluble says nothing (Fifth opinion, f. 192VA). According to a closely related view, 'no one can say that he himself says what is false, and there can be no proposition from which an insoluble can be generated' (Fourth opinion, f. 192VA). Spade points out a more sophisticated medieval version of *cassation*, which better approximates its modern counterpart (see 'Insolubilia' (footnote 6), 247). This version of *cassation* is discussed in a tract by Pseudo-Sherwood, edited by Marie-Louise Roure in 'La problématique des propositions insolubles au XIII<sup>e</sup> siècle et au début du XIV<sup>e</sup>, suivie de l'édition des traités de W. Shyreswood, W. Burleigh et Th. Bradwardine', *Archives d'histoire doctrinale et littéraire du moyen âge*, 37 (1971), 205-326; henceforth in references. 'Roure'. In 8.01-8.03, Pseudo-Sherwood presents three arguments providing semantic grounds for the claim that my utterance 'a falsehood is said' is not a proposition. Pseudo-Sherwood goes on to reject the arguments.

11 Paul writes: 'The eighth opinion posits that no insoluble is true or false, since no insoluble is a proposition. For although any insoluble is an indicative expression, and its signification signifies as is [the case] or as is not [the case], nevertheless this is not enough to warrant its being called a proposition' (f. 192VA).

12 Paul writes: 'The sixth opinion posits that an insoluble is neither true nor false, but a mean indifferent to each' (f. 192VA). According to Bradwardine, insolubles are neither true nor false, but have some intermediate value (see Spade, *A catalogue*, 107).

reference.<sup>13</sup> Further, it is arguable that there are medieval anticipations of Tarski's solution via a hierarchy of languages,<sup>14</sup> of Russell's type theory<sup>15</sup> and his vicious circle principle,<sup>16</sup> and of the view that Liar sentences are both true and false.<sup>17</sup>

In this paper, I examine one medieval resolution of the Liar. I shall argue that this resolution has not been adequately captured by modern commentators. Once it has been properly understood, we can forge a new link between it and certain modern solutions not in the above list. In this way, we provide further support for the claim that the medieval anticipation of modern treatments of the Liar is complete. But the interest of the medieval resolution goes beyond this, because though related to recent solutions, it is not reducible to them. This medieval line on the Liar, I shall argue, is original and of independent interest.

## 2. An outline of the Ockham-Burley-Pseudo-Sherwood solution

The medieval resolution in question is found in the writings of Ockham, Burley and Pseudo-Sherwood.<sup>18</sup> Though there are significant differences which I shall discuss later, the core of the approach is common to the three authors, and to this I now turn.

Each considers the following paradoxical situation.<sup>19</sup> Socrates says only 'Socrates says a falsehood'. From either assumption, that Socrates's utterance is true, or that it is false, we obtain a contradiction: this pair of arguments constitutes the two halves of the paradox.

To resolve it, all three authors invoke the rule of the *restringentes*: a part never

13 This is anticipated in the claim of the *restringentes*, that 'in no proposition does a part supposit for the whole of which it is a part'. The *restringentes*' claim is presented by Paul as a presupposition of the Fourteenth Opinion. I shall argue in sub-section 3.1 below that the rule of the *restringentes*, though it anticipates modern approaches which ban self-reference, does not itself constitute such an approach.

14 Ernest A. Moody raises this possibility in *Truth and consequences in medieval logic* (Amsterdam, North-Holland, 1953), 109. As we shall see, Roure finds such an anticipation in the approach of the *restringentes*, Pseudo-Sherwood and Burley. In Bocheński's opinion, Peter of Ailly accepts the distinction between language and metalanguage (see Bocheński (footnote 4), 249 and 35-44). Here, Bocheński follows Paul's account of Peter of Ailly.

15 Again, we will see that Roure argues this way. Several authors take Ockham's solution to be an application of his doctrine of impositions and intentions, and thereby an anticipation of Russell or Tarski. Spade, in his paper 'Ockham on terms of first and second imposition and intention, with remarks on the Liar paradox', *Vivarium*, 19 (1981), 47-55, cites the following: Philotheus Boehner, 'Ockham's theory of supposition and the notion of truth', in *Collected articles on Ockham* (ed. Elginus M. Buytaert: St. Bonaventure, N.Y., 1958), 232-267 (pp. 254-260); E. J. Ashworth, *Language and logic in the post-medieval period* (Dordrecht, Reidel, 1974), 104; Francesco Bottin, *Le antinomie semantiche nella logica medievale* (Padua, 1976), 173-175.

16 Bocheński attributes the idea of the vicious circle principle to Peter of Ailly (see Bocheński (footnote 4), 395). See Spade's *Peter of Ailly: concepts and insolubles* (Dordrecht, Reidel, 1980), par. 297, for the relevant passage.

17 The view that the Liar sentence is a true contradiction is attributed to Peter of Ailly (*ibid.*) by Laurence Goldstein in 'The paradox of the liar—a case of mistaken identity', *Analysis*, (1985), 9-13.

18 The relevant passages in Ockham are: *Summa logicae* III-3, Ch. 46; *Sophistical refutations*, Book II, Ch. 10, sec. 4 (OPh III, 267-268); and *Summa logicae* III-1, Ch. 4 (OPh I, 368). The tracts by Pseudo-Sherwood and Burley are edited in Roure. Martin Grabmann attributes Pseudo-Sherwood's tract to William of Sherwood. De Rijk argues that there is not sufficient evidence for the attribution. See Roure, 205; and Spade, *A catalogue*, 26. I mark this uncertainty by the label 'Pseudo-Sherwood'.

19 In Pseudo-Sherwood's example, Socrates' is replaced by 'I'. Both examples are found in Burley.

supposits for the whole of which it is a part.<sup>20</sup> Thus, the predicate 'falsehood' in Socrates's utterance does not supposit for Socrates's utterance, but for all other falsehoods. Following Pseudo-Sherwood's terminology, let the name 'A' stand for these other falsehoods. Then Socrates's utterance is equivalent to 'Socrates says a falsehood which is A'. Since, by hypothesis, Socrates says no other falsehood, what he says turns out to be straightforwardly false.

But the second half of the paradox leads from the falsity of Socrates's utterance to a contradiction. So this argument must be exposed as fallacious. Since the argument is differently presented by Pseudo-Sherwood on the one hand, and by Ockham and Burley on the other, and their respective treatments are not exactly analogous, I shall deal with each in turn.

Pseudo-Sherwood takes the problematic inference to be from "Socrates says a falsehood" is false' to 'Therefore, that Socrates says a falsehood is false'.<sup>21</sup> According to Pseudo-Sherwood, Socrates's utterance is to be understood as 'Socrates says a falsehood which is A', and since this is false, the premise is true. Someone who now fallaciously infers the conclusion makes the mistake of lifting the restriction on the supposition of the predicate 'falsehood' in Socrates's utterance. There is an unwarranted shift in the supposition of the predicate, from restricted to unrestricted. The error is to proceed

from an expression *secundum quid* or an expression with a determination to an expression *simpliciter*. And there is most straightforwardly [a paralogism] *secundum quid et simpliciter*.<sup>22</sup>

Ockham and Burley consider several versions of the fallacious argument. I shall take Ockham's treatment of one example as typical. Ockham writes:

... the inference 'Socrates says this falsehood; therefore Socrates says a falsehood' does not hold good, but there is a fallacy *secundum quid et simpliciter*. And this is because in the proposition 'Socrates says a falsehood', the term 'falsehood' cannot supposit for this falsehood—which is that Socrates says a falsehood.<sup>23</sup>

20 As we shall see, Ockham and Burley qualify this rule; Pseudo-Sherwood does not.

21 Rourke, 259–260, 11.01. As Pseudo-Sherwood presents it, this inference is the first of two steps leading from the assumption that Socrates says a falsehood to a contradiction. The second step is from 'That Socrates says a falsehood is false' to 'Socrates does not say a falsehood' (see Rourke, 249–250, 1.05).

22 In 11.01–11.03, Pseudo-Sherwood equivocates on the use of '*secundum quid*' and '*simpliciter*'. The terms are applied not only to the term 'falsehood' as it occurs within the argument, but also to (i) the dictum 'that I say a falsehood'; (ii) the term 'false' taken as a predicate of sentences, as in 'Socrates says a falsehood' is false'; and (iii) sayings, as in Socrates' saying 'Socrates says a falsehood'. This equivocation does not lead to any inconsistency, however. The application to the term 'falsehood' should be regarded as basic, and (i), (ii) and (iii) are derivative in obvious way.

23 Ockham, Oph. III, 267–268. The entire argument from the assumption that Socrates says a falsehood to a contradiction may be put this way: Suppose 'Socrates says a falsehood' is false. Socrates says this falsehood; therefore Socrates says a falsehood. So Socrates says what is true, and consequently Socrates says the truth. (Compare Ockham, *Summa logicae*, III–3, Ch. 46.)

A fallacy is committed by taking the predicate 'falsehood' in the conclusion to have unrestricted supposition when, in fact, its supposition is restricted. Like Pseudo-Sherwood, Ockham diagnoses the argument as a fallacy *secundum quid et simpliciter*.<sup>24</sup>

There are superficial differences between the two presentations. The argument presented by Pseudo-Sherwood is fallacious if the restriction does not apply in the conclusion, and valid if it does; for the argument presented by Ockham, the reverse is true. But all agree that there is a restriction on 'falsehood' in the conclusion, and in failing to notice this, we mistake a fallacious inference for a valid one.

The present paradox is generated by Socrates's sole utterance 'Socrates says a falsehood'. This proposition is characterized by Ockham and Burley as an *affirmative insoluble*. An example of a *negative insoluble* is Socrates's sole utterance 'Socrates does not say a truth'. Ockham and Burley treat this kind of insoluble in just the same way. Here, the predicate 'true' cannot supposit for Socrates's utterance. If we let 'A' stand for all other truths, what Socrates says is equivalent to 'Socrates does not say a truth which is A', which, by hypothesis, is true. Affirmative insolubles are false, but negative insolubles are true. The way in which Ockham and Burley deal with the corresponding fallacious arguments generated by negative insolubles follows the same pattern as before.<sup>25</sup>

3. An interpretation of the Ockham–Burley–Pseudo-Sherwood solution

We now have the bare bones of the Ockham–Burley–Pseudo-Sherwood solution. In this section my aim is to articulate it further. This I shall do by clearing up some misconceptions about it.

3.1 One such misconception is that the solution is essentially a ban on self-reference.<sup>26</sup> This interpretation rests on the assumption that the basis of the Ockham–Burley–Pseudo-Sherwood resolution is the rule of the *restringentes*: 'in no proposition does a part supposit for the whole of which it is a part'. In the case of Pseudo-Sherwood, this assumption is correct. Pseudo-Sherwood provides grounds for the *restringentes*' rule, in a fully unqualified form:

24 Ockham also takes insolubles to be fallacies *secundum quid et simpliciter* in *Summa logicae* III–1, Ch. 4; Oph. I, 368. The fallacy is not central to Burley's treatment. It is only briefly mentioned, in Rourke, 275, 4.04 and 4.05. In discussing the inference 'Socrates says this falsehood; therefore, Socrates says a falsehood', Burley suggests that to say this falsehood is to say *secundum quid* a falsehood. Burley writes: 'Thus the determination "*secundum quid*" should govern "to say" and not "falsehood"'. (4.04). In 4.05, Burley argues that if Socrates says that he says a falsehood, then, since he says the word 'falsehood', he *secundum quid* says falsehood (taken in material supposition). Thus Burley does not appeal to the fallacy in the same way as Ockham and Pseudo-Sherwood.

25 But, as we shall see in the next section, significant points arise from the details of their account of these arguments.

26 For example, Paul Spade and E. J. Ashworth encourage such a view. See Spade, 'Ockham on self-reference', *Notre Dame Journal of Formal Logic*, 15 (1974), 298–300; *A catalogue*, 112, 119–120; 'Insolubilia' (footnote 6), 248 and fn. 25 and 26; review of E. J. Ashworth (footnote 15), in *Dialogue*, 15 (1976), 337–338; 'Ockham' (footnote 15), 34–55. See also Ashworth (footnote 15), 104.

As they say, it seems that the part cannot support for the whole. For when the expression is argued [with] in the intellect, the understanding of the predicate and [the understanding] of the subject are first taken separately and afterwards put together [in a sentence or proposition]. And they are put together in the proposition in the way they were taken [at first]. But the expression 'falsehood' was not taken for the whole, because this composite expression did not exist in the intellect then. Therefore 'falsehood' does not enter into the composition [insofar as it is taken] for this.<sup>27</sup>

Now the argument presented here applies not only to the term 'falsehood' as it occurs in Socrates's utterance 'Socrates says a falsehood', but also to the term 'falsehood' as it occurs in the sentence

(S) '2 + 2 = 4' is a falsehood',

or the term 'sentence' as it occurs in the sentence

(S) '2 + 2 = 4' is a sentence'.

If we apply the restriction to Socrates's paradoxical utterance, then what Socrates is saying is 'Socrates says a falsehood which is A'. So Socrates does not succeed in talking about his very utterance, but only about other falsehoods. It may be plausible to construe this as ruling out self-reference. But there is *nothing* self-referential about the sentences (S) and (S'); and yet the *restringentes* rule applies. That is, the rule places restrictions within sentences that are not in any sense self-referential. It bans self-reference only in a highly attenuated way: words like 'falsehood' and 'sentence' cannot 'refer' to the very sentences in which they occur. And this restriction applies irrespective of whether or not the sentences themselves are self-referential. So, unlike modern approaches which ban self-reference, Pseudo-Sherwood's resolution does not diagnose self-referential sentences as semantically pathological because of their self-referential nature. Rather, the treatment of such sentences arises as a consequence of a more general rule; and to describe this rule as a ban on self-reference is strained.

Unlike Pseudo-Sherwood and the *restringentes*, Ockham and Burley limit the scope of the *restringentes* rule to the insolubles only.<sup>28</sup> So at first glance it may seem

27 Rourke, 251, 3.02. Here in 3.02, Pseudo-Sherwood sets out the *restringentes* argument. In 3.03, he argues further along similar lines. In 4.05-4.08, he defends the *restringentes* against various attacks laid out in 4.01-4.04. In 4.10, he clearly endorses the *restringentes* rule, and in 1.1.02 he defends its use by the argument of 3.02.

So, Paul Spade is mistaken when he writes of Pseudo-Sherwood: "The opinion of the *restringentes* is also rejected (3.01-4.10) . . . The author replies (4.01-4.10) by setting out unproblematic instances of self-reference, and by distinguishing between an integral whole and a universal whole" (*A catalogue*, 27). The objections to the *restringentes* which Spade cites occur in 4.02 and 4.04; Pseudo-Sherwood replies to them in 4.06 and 4.08 respectively. Pseudo-Sherwood objects not to the *restringentes* rule but to their treatment of the fallacious argument (for details, see 4.10).

28 In 2.05 and 2.06, Burley argues against the *restringentes* rule, and in 3.03 states the restricted version.

plausible to characterize their solution as a ban on self-reference. But such a characterization is demonstrably mistaken. For it is clear from their treatment of negative insolubles that the restrictions which Ockham and Burley place on supposition do not always coincide with applications of the limited *restringentes* rule. Burley writes:

Nor does the following inference hold: 'That a proposition spoken by Socrates is not true is true; and that a proposition spoken by Socrates is not true is a proposition spoken by Socrates; therefore a proposition spoken by Socrates is true'. For the term 'proposition spoken by Socrates' in the conclusion cannot support for the whole 'a proposition spoken by Socrates is not true'. And the term does support for this in the minor. Therefore, the inference does not hold good.<sup>29</sup>

The crucial point to notice is that the proposition for which the term in the conclusion cannot support is different from the one of which the term is a part. The restriction here is *not* according to the rule of the *restringentes*.<sup>30</sup>

What rule *does* this restriction accord with? Although Burley appeals to the *restringentes* rule, he also suggests another.<sup>31</sup> This says that the restriction on supposition applies wherever its violation leads to something 'self-reflexive with a privative determination'. For example, consider the affirmative insoluble 'I say a falsehood'. Suppose we let the term 'falsehood' support for my utterance. When we replace the term by this suppositum, we obtain 'I say that I say a falsehood'. But this is self-reflexive with a privative determination, since '[h]ere the act of speaking reflects back over the act of speaking with the determination "falsehood", which is a privative determination'.<sup>32</sup> In the case of fallacies associated with affirmative insolubles, this

In *Summa logicae* III-3, Ch. 46, Ockham qualifies the *restringentes*'s rule. Spade argues persuasively that Ockham rejects the *restringentes*' rule in all but exceptional cases, and that these would seem to be the insolubles (see 'Ockham on Self-Reference' (footnote 26)).

29 Rourke, 273, 3.07.

30 There are further examples of this in Burley's work. In 4.05, he considers the following case: Socrates says only 'Socrates says a falsehood', and Plato says 'Socrates does not say a falsehood'. Burley argues that the following inference does not hold (where the reference of the demonstrative 'this' is Plato's utterance): 'A contradictory of *this* proposition is said by Socrates; therefore the contradictory of a proposition is said by Socrates'. Applying Burley's rule in the natural way, 'proposition' in the conclusion cannot support for Plato's utterance. In 4.08, in the course of discussing an inference presented in 1.08, Burley writes: ". . . For the term "antecedent" in "Socrates draws an inference from an antecedent" cannot support for "Socrates draws an inference from a falsehood" . . . . The treatment of these cases suggests a way in which the Ockham-Burley-Pseudo-Sherwood line might deal with *langtes* (e.g. Socrates says only 'Plato says a falsehood', and Plato says only 'Socrates says a falsehood') and *chaintes* (e.g. (1) (2) is false; (2) (3) is false; (3) (4) is false; . . .).

31 In just one place (3.03), Burley treats his rule as limiting that of the *restringentes*. Elsewhere (3.02, 4.01, 4.03, 4.06, 4.09) he cites his rule and makes no mention of the *restringentes*. He does not provide a precise characterization of his rule: it is not clear exactly what counts as something self-reflexive with a privative determination. But what is clear is that Burley is operating with some general rule which does not coincide with that of the *restringentes*.

32 Rourke, 272, 3.02.

rule and the *restringentes*, coincide in their results. But they pull apart in the case of fallacies associated with negative insolubles. In Burley's example, if we ignore his rule, we obtain 'That a proposition spoken by Socrates is not true is true'. Socrates's assertion of this may be taken to be self-reflexive with a privative determination. The restriction Burley places on supposition here is in accordance with his rule, *not* with that of the *restringentes*.

Consider now Ockham's corresponding argument. He writes:

... just as the inference 'This is true, and Socrates says it, therefore he says a true proposition other than this' does not hold good, so the inference 'Socrates says the proposition "Socrates does not say a truth", and this is true; therefore Socrates says a truth' does not hold.<sup>33</sup>

The natural way to make out the parallel between these inferences is to restrict the supposition of the term 'truth' in the conclusion of the second inference, so that it cannot support for the proposition 'Socrates does not say a truth'. Again, it is *not* the *restringentes* rule that is being applied here. If we replace the term 'truth' in the conclusion by Socrates' utterance, we obtain 'Socrates says that Socrates does not say a truth', which is self-reflexive with a privative determination. Perhaps, then, Ockham is implicitly applying a rule like Burley's—but whatever the rationale, it is not one provided by the *restringentes*.

So Ockham and Burley do not resolve the Liar by a ban on self-reference. Indeed, there is reason to suppose that they would admit directly self-referential sentences. Suppose I say 'This very sentence is a falsehood'. Here is the natural way to apply the Ockham-Burley-Pseudo-Sherwood resolution to this example: the predicate 'falsehood' in my utterance cannot support for my utterance, which is then equivalent to 'This very sentence is a falsehood which is A', where 'A' stands for all other falsehoods. But now what I say is clearly false, since my utterance is identified with some falsehood other than it.

Nevertheless, in saying what I do, I succeed in talking about my very utterance: I say something false about it. Here, then, there is no problem in my referring to my very utterance. Of course, according to the *restringentes*, and Pseudo-Sherwood, the subject term 'this very sentence' cannot support for my utterance. But this further restriction plays no role in resolving the paradox, and so Ockham and Burley would have no reason to invoke it.<sup>34</sup>

3.2 A second misconception about the medieval resolution concerns the claim made by Pseudo-Sherwood and Ockham that the Liar is correctly diagnosed as involving a fallacy *secundum quid et simpliciter*. The thought is that, under the influence of

33 Ockham, *Summa logicae*, III-3, ch. 46; OPH I, 745.

34 The point made in this paragraph can be extended to quantifiers. Suppose Socrates says only 'Everything Socrates says is a falsehood'. Again, the natural way to apply the Ockham-Burley-Pseudo-Sherwood solution to this case is to restrict the supposition of 'falsehood', so that what Socrates says is equivalent to 'Everything Socrates says is a falsehood which is A', which is a falsehood. No restriction is placed on the quantifier.

Aristotle, medieval authors forced insolubles into a pattern which they do not fit. Paul Spade argues that all medieval solutions along these lines are strained.<sup>35</sup> Marie-Louise Rourke claims that Pseudo-Sherwood does not completely succeed in placing his solution within the framework of the fallacy, and even suggests that he may not have been seriously committed to this classification.<sup>36</sup>

The fallacy *secundum quid et simpliciter* arises when an expression used in a restricted way is taken as if it were unrestricted. A leading example of Aristotle is the inference from 'The Ethiopian is white, with respect to his teeth and eyeballs' to 'The Ethiopian is white, absolutely'. In fallaciously drawing this conclusion, we neglect a restriction attaching to the term 'white': the term 'white' applies to parts of the Ethiopian, but not to the Ethiopian as a whole. The part/whole distinction here, to put it in medieval terms, is between an integral part and an integral whole.<sup>37</sup> And if this is the example we have in mind, it is unclear how the Liar argument is an instance of this fallacy.<sup>38</sup> What is the integral part, and what is the integral whole? To fix ideas, take the fallacy associated with the affirmative insoluble 'I say a falsehood'. According to Pseudo-Sherwood, Ockham and Burley, some occurrences of 'falsehood' within the argument have unrestricted supposition, while others support for falsehoods other than Socrates's utterance. That is, the part/whole distinction is between some of the falsehoods and all of the falsehoods: in the terminology of certain medieval authors, including Burley, the distinction is between a universal part and a universal whole.<sup>39</sup> The question, then, is whether *this* part/whole distinction has a place in fallacies *secundum quid et simpliciter*.

From the twelfth century on, medieval authors distinguished various forms of the fallacy.<sup>40</sup> The example of the Ethiopian suggests one form, but it is not the only one. Ockham writes:

35 See Spade's 'Insolubilia' (footnote 6), 247; and his introduction to *Peter of Ailly* (footnote 16), 4. He is surely right about many medieval solutions of this kind.

36 See Rourke, 229-230: "La solution de Shyreswood qui s'insère, ou prétend s'insérer, dans le cadre de la solution *per secundum quid et simpliciter* . . . ."

37 Broadly speaking, on the medieval view an integral whole is a composite entity, and an integral part is obtained by physical division of the whole. A stock example of an integral whole is a house, which has walls, roof, and foundations as its parts; another example of an integral whole is an individual man. (For a discussion of this, see Norman Kretzmann's 'Syncategoremata, expomibilia, sophismata', ch. 11 of *The Cambridge history* (footnote 6), 230ff, esp. fn. 77).

38 And it seems that this is the example Spade and Rourke have in mind. It is the only example they mention in discussing the fallacy and medieval solutions to the Liar. See Spade's *Peter of Ailly* (footnote 16), 4, and Rourke, 226.

39 The universal whole is taken as an aggregate, rather than an abstract object like a set. Part of the universal whole composed of all the men is all the Caucasians.

40 See Norman Kretzmann, in William of Sherwood's *Introduction to logic* (Minneapolis, University of Minnesota Press, 1966), 153-154, fn. 124, where Kretzmann distinguishes 'four or more' kinds of determination in a certain respect (or 'unstrict determination') which are to be found in L. M. De Rijk, *Logica modernorum: A contribution to the history of early terminist logic* (Van Gorcum, 1962). One example Kretzmann cites, the inference from 'a chimera is thinkable' to 'a chimera is', is particularly relevant to our considerations below. Some of these early authors discuss both the fallacy and ampliation (see, for example, the tract *Dialectica Monacensis*, in De Rijk, Vol. 2, Part 2, 626-630, 670-671), but none connected the two.

Nor is the fallacy of inferring an absolute from a relative predication always made by arguing from a determinable taken together with a determination that diminishes [the supposition] to that determination taken absolutely.<sup>41</sup> But that form of argument is only one form that is fallacious in this way. And another form occurs when the argument is from a proposition having a predicate to a proposition of the second adjacent, as for example in the argument 'Socrates is believed in, therefore Socrates exists'.<sup>42</sup>

And Albert of Saxony writes:

... I maintain that there are two principal ways to commit this fallacy. The first occurs when there is an argument from the third adjacent to the second adjacent in the affirmative, or from the third adjacent to the second adjacent in the negative. Example of the first: 'The Antichrist is conceivable; therefore the Antichrist is...'<sup>43</sup>

These examples are clearly suggested by Aristotle, at the very outset of his discussion of the fallacy:

Fallacies connected with the use of some expression absolutely or in a certain respect and not in its proper sense, occur when that which is predicated in part only is taken as though it was predicated absolutely. For example, 'If that-which-is-not is an object of opinion, then that-which-is-not is'; for it is not the same thing 'to be something' and 'to be' absolutely.<sup>44</sup>

The examples, then, are unquestionably genuine instances of the fallacy *secundum quid et simpliciter*. Now what we want to know is how they are analysed by Ockham and Albert.

The answer is found in their theory of *ampliation*. These examples of the fallacy are treated as cases involving amplified supposition. Ockham maintains that an argument such as 'The Antichrist is not; therefore the Antichrist is not possible'

moves from 'is' taken *per se* to ['is'] taken together with a predicate after it, where the predicate is such as to be able to pertain to a term that suppositis for a non-being as well as for a being. Whenever the argument goes this way, there is a fallacy *secundum quid et simpliciter* when one argues in the negative, and conversely when one argues in the affirmative. For example, these [inferences]—'a is producible by God; therefore a is' and 'a is not therefore a is not producible by God'—contain a fallacy *secundum quid et simpliciter*. Likewise, the [inference] 'The Antichrist is not; therefore the Antichrist is not foreknown by God', and conversely 'The Antichrist is foreknown by God; therefore the Antichrist is'.

41 The case of the Ethiopian is an example of this form of the fallacy.

42 Ockham, Ord. I, d.2, q.7; OTh II, 246. See also *Summa logicae* III-4, ch. 13.

43 Albert of Saxony, *Perutilis logica* (Venice, 1522), Treatise V, ch. ix, Article iii.

44 Aristotle, *On sophistical refutations* (footnote 3), 166b38-167a4.

Thus, since what is not can be foreknown as much as what is, there is a fallacy *secundum quid et simpliciter* in such [inferences].<sup>45</sup>

So in the sentence 'a is', for example, the copula allows the subject term to range over only presently existing things. In the sentence 'a is producible by God', the subject term ranges not only over presently existing things, but over all possibly existing things, since it is for these that the predicate 'producible by God' stands. The predicate has the effect of ampliating or extending the supposition of terms to which it is adjoined. This is the case with modal terms in general, and with verbs in the past and future tense. In relation to a present tense copula in a proposition of the second adjacent, the subject term suppositis only for presently existing things. In relation to a past-tense or future-tense copula, or a verb of possibility (e.g. 'can'), or predicates that can supposit not only for beings but also for non-beings (things that do not exist in the present), the subject term can supposit for past, future or possible things as well.

Now we can diagnose the fallacious inference from 'a is producible by God' to 'a is'. What may escape us is that there is a shift in the supposition of the subject term from all possible things to all presently existing things. We are prone to take the subject term in the conclusion to supposit for a universal whole (all possible things), when in fact it ranges over a part of that universal whole (those possible things which are presently existing).<sup>46</sup>

45 Ockham, *Summa logicae* III-4, ch. 13. Ockham discusses the ampliating effect of verbs in the past and future tense in *Summa logicae* II, ch. 7, 269. For Albert of Saxony's theory, see *Perutilis logica* (footnote 43), Tract Two, ch. X. Albert's Seventh Rule for Ampliation says: 'When a proposition has a present-tense copula but a predicate that includes the verb "can" — such as verbal nouns that end in "able" — the subject is amplified to supposit for what is or for what can be'. Other rules cover verbs in the past and future tense.

There is a common medieval account. See also Paul of Venice, *Logica magna*, Part I 'On the suppositio of terms', fol. 25VA, lin. 21-45, 25VB, lin. 38-72 (ed.). This theory takes the range of the supposition of a term to be presently existing things, which may be amplified by certain adjoining terms. According to an alternative account (see, for example, Peter of Spain, *Tractatus called afterwards Summulae logicae* (ed. L. M. De Rijk; Assen, 1972), 209, 4-10), a term by itself suppositis for all things, past present or future, but its range of supposition may be restricted under special conditions. (For a discussion of this, see L. M. de Rijk's 'The origins of the theory of the properties of terms', ch. 7 of *The Cambridge history* (footnote 6), 161-173, (pp. 171-172). On either view, there is a shift in supposition between a universal whole and a universal part.

46 Another fallacious inference which Ockham treats in the same way is 'Where no rose exists, a rose is understood; therefore, a rose exists'. The term 'rose' here has determinate personal supposition, and it is clear how this kind of supposition may be widened. Ockham and Albert do not explain how their theory of ampliation works for terms like 'the Antichrist' or 'Socrates'. These terms have *discrete* supposition, and it is unclear how this kind of supposition may be widened. Perhaps they would find the following line agreeable. The sentences 'A rose exists' and 'The Antichrist is' each are true if and only if there is a presently existing suppositum of the subject term. The sentences 'A rose is understood' and 'The Antichrist is conceivable' each are true if and only if there is a presently existing suppositum of the subject term, or a thinkable which is a suppositum of the subject term; and here, in each case, there is a widening of the range of things from which supposita can be drawn. The only difference between the cases is the *number* of supposita. There are many supposita for 'rose' in both its occurrences; there are no supposita for 'the Antichrist' in the first of its occurrences, and just one in the second.

But this is just how things are with the Liar, according to Ockham, Burley and Pseudo-Sherwood. When we illegitimately infer the conclusion, we take 'falsehood' to support for a universal whole (all the falsehoods), when in fact it supports for a universal part (some of the falsehoods). Again, there is a term in the conclusion of the fallacious argument which supports for only a part of the universal whole for which we think it stands. There is, then, no reason to deny that this is indeed a genuine form of the fallacy *secundum quid et simpliciter*, or that Pseudo-Sherwood and Ockham took it this way.

3.3 A third misconception draws a false analogy between this medieval solution and the modern approaches of Russell and Tarski. Several authors treat Ockham's solution in a way which suggests an anticipation of Russell's theory of types or Tarski's hierarchy of languages.<sup>47</sup> According to Marie-Louise Roure, the approaches taken by Pseudo-Sherwood, Burley and the *restringentes* also bear a resemblance to the theories of Russell and Tarski.<sup>48</sup> I shall focus on Roure's interpretation of Pseudo-Sherwood, Burley and the *restringentes*, but what I have to say applies equally well to the corresponding interpretation of Ockham.

Roure argues that in the solutions of Pseudo-Sherwood, Burley and the *restringentes*, we can discern an implicit distinction between two levels, that of the discourse, and that of discourse about the discourse.<sup>49</sup> At the level of the discourse, there is the insoluble itself; for example, Socrates's sole utterance, 'Socrates says a falsehood'. This utterance is false, given the restriction on the term 'falsehood'. At the second level of discourse, we reflect on his utterance. We ask whether, in saying what he did, Socrates says a truth or a falsehood.<sup>50</sup> As we have seen, from the assumption that Socrates' utterance is false, an apparent contradiction is obtained, in a variety of

ways. These problematic inferences belong to the level of discourse about the discourse: they arise from reflection on the truth value of what Socrates says. Such an inference, Roure concludes, constitutes a paradox at this second level, which is solved by distinguishing restricted and unrestricted occurrences of 'falsehood' within the inference.<sup>51</sup>

I do not find Roure's two-level interpretation plausible. It is true that the medieval line is concerned with two distinct pieces of discourse. There is Socrates's original utterance, 'Socrates says a falsehood'. And then there is the argument which leads from the falsity of his utterance to a contradiction; for example: 'Socrates says a falsehood' is a falsehood, and Socrates says a falsehood; therefore Socrates says a falsehood'.<sup>52</sup> Of course, the argument is *about* Socrates's utterance, and so, in an uninteresting way, it is a "metadiscourse".<sup>53</sup> The question is whether it is in any sense a *Tarskian* metadiscourse.

Tarski showed that no classical formal language contains its own truth predicate. The truth predicate for such a language (the object language) is contained in a metalanguage. And the truth predicate for this metalanguage is contained in a further metalanguage: in this way, an infinite hierarchy of formal languages is generated, and with it, infinitely many *distinct* truth predicates.

Now the key difference between our discourse and "metadiscourse" is that in the argument there is an unrestricted occurrence of 'falsehood', whereas in Socrates's utterance there is only a restricted occurrence. This may tempt us into drawing a distinction between two falsity predicates, 'false (which is A)' and 'false (unrestricted)', and correlating different levels of language with these predicates, so that our "metadiscourse" is a Tarskian metadiscourse.<sup>54</sup> But this temptation should be resisted. For in the argument, there are *restricted*, as well as unrestricted, occurrences of 'falsehood'. (This is precisely why, according to Ockham, Burley and Pseudo-

from what he says, that he says a truth or that he says a falsehood. This, I think, is how to take Burley's discussion, which occurs in 3.04. There is nothing here which suggests a distinction of levels of language.

As Roure points out (p. 234, fn. 2), Ockham makes an exactly similar claim. But with it comes an elaboration which clearly rules out Roure's interpretation: 'For when Socrates begins to speak by saying "Socrates says a falsehood", and one asks whether Socrates says a truth or a falsehood, it must be replied that Socrates says neither a truth or a falsehood, *just as it must be granted that he neither says a truth nor a falsehood other than this [proposition] . . .*' (*Summa logicae*, III-3, ch. 46; Oph I, 746, my emphasis). For Ockham, the sense in which Socrates says neither a truth nor a falsehood is this: Socrates says neither a truth nor a falsehood which is A.

51 The *restringentes* proceed differently, diagnosing the argument either as a fallacy *figuram dictionis* (see Bradwardine, 5.01), and Pseudo-Sherwood, 3.01; it is this part of the *restringentes'* solution to which Pseudo-Sherwood is opposed), or as a fallacy *non causam ut causam* (see Bradwardine, 5.03).

52 Here we follow an example of the kind suggested by Ockham, Burley and the *restringentes*. The remarks that follow apply equally well to Pseudo-Sherwood.

53 In 10.08-10.11, Pseudo-Sherwood, in the course of arguing that there is no problem in talking about propositions or falsehoods, gives no suggestion of a distinction of levels of language.

54 Such an account is 'Tarskian' in the sense that it treats natural languages the way that Tarski treats formal languages. But Tarski himself does *not* endorse such accounts of natural language; see, for example, 'The concept of truth in formalized languages', in *Logic, semantics, metamathematics* (2nd ed. Hackett, Philadelphia, 1983), 152-278 (p. 267).

47 See footnote 15. In his article cited there, Spade points out that (i) Ockham nowhere appeals to his doctrine of impositions and intentions in his discussion of the Liar, and (ii) the plausibility of such an anticipation is undermined by Ockham's adoption of the *restringentes'* rule *only* in exceptional circumstances.

48 Roure, 224-25, links the *restringentes'* solution with Russell's theory of types. She would presumably extend such links to Pseudo-Sherwood and Burley, since she finds no real difference between Pseudo-Sherwood and the *restringentes* (p. 230), and finds Burley to be the most authentic of the *restringentes* (p. 234). (See also the quotation given below in this footnote.) On p. 234, she ascribes to Burley implicit recognition of the distinction between levels of language as a way of resolving the paradox. On p. 244, she raises the question as to whether there are resemblances between medieval and modern solutions to the Liar, and answers this question in the affirmative on pp. 246-247, arguing that the appeal to two levels of discourse as a way of solving the paradox 'n'est pas absolument inconnu des logiciens médiévaux, encore qu'il ne soit pas toujours explicite. Nous l'avons rencontré à diverses reprises au cours de notre étude; notamment chez certains "restringentes", ainsi que chez Burleigh et Shyreswood'.

On p. 234, fn. 2, Roure links Burley's solution to Ockham's; there is every reason to suppose that she would carry over her analysis to Ockham.

49 Roure, 222 (concerning the *restringentes*), 229 (concerning Pseudo-Sherwood), 232 (concerning Burley).

50 According to Roure's interpretation on p. 232, Burley claims that at this second level, Socrates says *neither a truth nor a falsehood*. She takes this as evidence that Burley implicitly resists his solution on a distinction of language levels (p. 234). I suggest a different interpretation. It follows from Burley's analysis that Socrates does not say anything *about* his utterance. In saying what he does, Socrates does not say that what he says is true or false. If Socrates says that he says a falsehood, we cannot infer, just

Sherwood, such arguments are fallacies.) So, if we distinguish falsity predicates in the way suggested, there would be occurrences of both within a single discourse, and this is not possible on the Tarskian account. The attempt to treat the argument as a Tarskian metadiscourse assumes that it has some definite metal level, to which the uses of 'falsehood' correspond; but the attempt leads to the absurd conclusion that the argument belongs at once to distinct levels, the metal level and the object level. The upshot is that the relation here between "metadiscourse" and discourse bears no resemblance to the relation between a Tarskian metalanguage and object language.

Ockham, Burley and Pseudo-Sherwood have *no need* of a distinction between levels of discourse. They proceed by placing restrictions on certain occurrences of the semantical predicates,<sup>55</sup> and by this alone. This suggests the correct view of their solution: it is an *indexical* solution. It treats the semantical predicates 'true' and 'false' as *indexicals*; that is, as terms whose linguistic meaning is fixed, while their extensions vary according to the context of utterance. (Compare 'I', 'you', 'here', 'now'.) The idea is not that there are different truth (and falsity) predicates, one for each level of discourse. Rather it is that there is a single truth predicate, and a single falsity predicate, the extensions of which vary according to context.

Pseudo-Sherwood gives particularly clear expression to the indexical idea, in responding to this objection to the *restringentes*:

Again, the restricters say that this—that I say a falsehood—is false. Therefore, they grant that this is a supposition of 'falsehood'. And so 'falsehood' suppositis for this, even though it is a part of it.<sup>56</sup>

Pseudo-Sherwood defends the *restringentes*, and at the same time his own view, as follows:

In reply to another it must be said that the restricter, when he replies 'a falsehood exists', already the term 'falsehood' in his reply is in another piece of discourse from the one proposed. And he is right to grant that the term 'falsehood' which is in another piece of discourse could supposit for the former one [viz. the first instance of 'I say a falsehood'] but the term 'falsehood' in that piece of discourse [can]not.<sup>57</sup>

We find a statement to similar effect in Ockham (where the inference to which he refers is 'Socrates says this falsehood; therefore Socrates says a falsehood'):

All insolubles should be replied to this way: viz., by denying the inference from a term taken with a demonstrative pronoun that indicates something contained [in what he said] to the term taken without such a pronoun. The only reason is that a

55 As we have seen in the case of Burley, it is not only the semantical predicates that are so treated: see footnote 30 and text.  
 56 Roure, 252, 4.03.  
 57 Roure, 253, 4.07.

common term cannot supposit in that proposition for that inferior, although it could supposit for it in other propositions.<sup>58</sup>

And Burley's general principle expresses the indexical idea, since it restricts certain occurrences of a term and not others, according to context.

Indexicality is deeply embedded in the medieval theory of supposition, and the parameters of indexicality go beyond the modern ones. We can distinguish two levels at which indexicality is treated in medieval supposition theory. At the first level of proposition types, there are general principles governing restrictions on supposition. The operative extension of any (singular or predicate) term depends on the propositional context. Parameters of the propositional context are the quantifier, negation, the tense of the verb, and ampliating terms. (One principle may tell us that in a certain proposition, a given term has confused and distributive supposition; another may tell us that an unampliated term suppositis for presently existing things.) At the second level of proposition tokens, the environmental context comes into play, and this will include the modern parameters of speaker, time, and place. At this level, a definite extension can be attributed to a token of a term. (For example, the extension may be all things existing at a specific time.) Principles like Burley's and the *restringentes* belong to the first level. At the second level, tokens of 'true' and 'false' have definite extensions according to the particular context of utterance and the rules of the first level.

Thus far, what picture do we have of the Ockham-Burley-Pseudo-Sherwood solution? It is a non-hierarchical, indexical solution. It is not to be identified with an approach which bans self-reference. It diagnoses liar arguments as fallacies which are appropriately placed in the framework *secundum quid et simpliciter*. In the final section, I fill out this picture some more, by exploring the connections between this medieval solution and certain modern proposals.

4. Comparisons and contrasts with some modern approaches

In his paper 'Russell's mathematical logic', Gödel notes that Russell's theory of types brings in a new idea for the solution of the paradoxes:

It consists in blaming the paradoxes . . . on the assumption that every concept gives a meaningful proposition, if asserted for any arbitrary object or objects as arguments.<sup>59</sup>

Gödel goes on to say that the simple theory of types carries through this idea on the basis of a further assumption, that whenever an object *x* can replace another object *y* in a proposition, and preserve meaningfulness, it can do so in any meaningful proposition. This means that objects are grouped into mutually exclusive ranges of

58 Ockham, *Sophistical refutations*, Book 2, ch. 10 sec. 4; OPh III, 267-268.  
 59 Kurt Gödel, 'Russell's mathematical logic', in P. A. Schilpp (ed.), *The philosophy of Bertrand Russell* (La Salle, Open Court, 1944), 123-153 (p. 149).



significance; each concept is significant only for arguments taken from one such range. In Russell's simple theory of types, each concept is significant for an infinitely small portion of all objects.

Gödel finds this further assumption unacceptable.<sup>60</sup> His suggestion is to reject it, while retaining the idea that not every concept gives a meaningful proposition for any object as argument.

It is not impossible that the idea of limited ranges of significance could be carried out without the above restrictive principle. It might even turn out that it is possible to assume every concept to be significant everywhere except for certain 'singular points' or 'limiting points', so that the paradoxes would appear as something analogous to dividing by zero. Such a system would be most satisfying in the following respect: our logical intuitions would then remain correct up to certain minor corrections, i.e. they could then be considered to give an essentially correct, only somewhat 'blurred', picture of the real state of affairs.<sup>61</sup>

Gödel does not enlarge upon this idea. It is helpful to do so by way of an example. Consider the *heterological* paradox. Call those words which do not have the property which they denote *heterological*. Thus the words 'long', 'alive', 'ambiguous' are heterological: they are not true of themselves. A paradox arises when we ask whether 'heterological' is heterological. Following through Gödel's suggestion, we suppose that the concept of heterologicality is significant for any object as argument, except for certain 'singular points'. One such singular point is the expression 'heterological': if it were not a singular point, the sentence "'heterological' is heterological" would generate a paradox. According to Gödel's solution, the sentence "'heterological' is heterological" would generate a paradox. According to Gödel's solution, the sentence "'heterological' is heterological" would generate a paradox.

Gödel's hope is for a non-hierarchical solution to the paradoxes. In the case of the Liar, the idea is that we locate singularities in our semantical concepts, rather than distinguish levels of language in a hierarchy. Semantical predicates are *univocal*, and are not subject to Russellian typical ambiguity. There is just one word 'true' in English, not a hierarchy of distinct predicates 'true<sub>1</sub>', (the subscript indicating level). Paradox is avoided by a *minimal* restriction on the range of significance of 'true' or predicates constructed from it (like 'heterological').

60 Gödel raises two objections (*ibid.*, 149). 'What makes the above principle particularly suspect, however, is that its very assumptions makes its formulation as a meaningful proposition impossible, because  $x$  and  $y$  must then be confined to definite ranges of significance which are either the same or different, and in both cases the statement does not express the principle or even part of it. Another consequence is that the fact that an object  $x$  is (or is not) of a given type also cannot be expressed by a meaningful proposition.'

61 *Ibid.*, 150.

62 Gödel's idea is developed along these lines by J. F. Thomson, 'On some paradoxes', in R. J. Butler (ed.), *Analytical Philosophy*, Series I (Oxford, 1962), 104-119. See also Alonzo Church, 'A set of postulates for the foundation of logic', *Annals of Mathematics*, 33 (1932), 346-366, 34 (1933), 839-864; Jaakko Hintikka, 'Identity, variables and impredicative definitions', *Journal of symbolic logic*, 21 (1956), 225-245 (esp. p. 241), and 'The vicious circle principle and the paradoxes', *Journal of symbolic logic*, 22 (1957), 245-249; and Hans G. Herzberger, 'New paradoxes for old', *Proceedings of the Aristotelian Society*, (1980-1981) 109-123 (esp. pp. 115-118).

In certain ways, the Ockham-Burley-Pseudo-Sherwood resolution is very much in the spirit of Gödel's suggestion. Like Gödel, Ockham, Burley, and Pseudo-Sherwood aim for a 'single-level', non-hierarchical solution to the Liar. And, to achieve this, *minimal* restrictions are placed on the extensions of semantical predicates, so that in our fallacious reasoning we are minimally incorrect.

There are disanalogies, too. Ockham, Burley and Pseudo-Sherwood place minimal restrictions on *tokens* of semantical predicates; according to the natural way of developing Gödel's idea, it is semantical predicate *types*, denoting semantical concepts, that have singularities. For Gödel, the Liar demonstrates that there are *systematic*, though minor, errors in our understanding of semantical concepts. For Ockham, Burley and Pseudo-Sherwood, the Liar does not reveal an imperfect grasp of semantical concepts, but rather a minimal misunderstanding about what the concept-word picks out on certain occasions of use. Our mistake is relative to the context of utterance: we do not go wrong in any *principled* way. Compare the competent language user who understands the linguistic meaning of the indexical 'here', but who, on some particular occasion of use, makes a mistake about what the term picks out, because the empirical circumstances so conspire.

There is a further contrast between the views. A Gödelian singularity does not belong to the *range of significance* of the relevant predicate. But Ockham, Burley and Pseudo-Sherwood restrict only the *extension* of the predicate: there is nothing outside the range of significance of *any* predicate. For example, the affirmative insoluble 'Socrates says a falsehood' is in the range of significance of the token of 'falsehood' occurring within it; the token is false of it. For Gödel, paradoxical sentences, like 'Heterological' is heterological, are meaningless and have no truth value. But for Ockham, Burley and Pseudo-Sherwood, insolubles are perfectly meaningful, and are either true or false.

Gödel's solution faces a problem which is sometimes called the Strengthened Liar. If the paradoxical sentence, say "'Heterological' is heterological", is meaningless, then it is not true. Thus the term 'heterological', when predicated of itself, yields a sentence which is not true. But then, 'heterological' is heterological. And we have asserted the very sentence which we originally argued was not true: it seems that it is true after all. So Gödel's claim that the paradoxical sentence is meaningless does not prevent paradox.<sup>63</sup> The Strengthened Liar also demonstrates the inadequacy of truth-value gap approaches to the Liar. We may avoid a contradiction if we claim that a Liar sentence like 'This sentence is false' is neither true nor false (which we may wish to distinguish from being meaningless); but this appeal to truth gaps only gives rise to 'strengthened' versions of the Liar sentence, like 'This sentence is either false or neither true nor false'.

The Ockham-Burley-Pseudo-Sherwood line, however, makes no appeal to meaningfulness, or to truth value gaps, and so does not fall foul of the Strengthened Liar. It operates with a fully classical bivalent semantics: *all* sentences, *including* the

63 Gödel writes: 'The obvious objection that every concept can be extended to all arguments, by defining another one which gives a false proposition whenever the original one was meaningless, can easily be dealt with by pointing out that the concept ('meaningfully applicable') need not itself be always meaningfully applicable' (footnote 59), 149). But this point will not meet the argument just given.

insolubles, are either true or false. The evaluation of insolubles as true or as false leads not to the Strengthened Liar, but to other problematic inferences (see section 2 and sub-section 3.1 above). But these present no new difficulties. As we have seen, Ockham, Burley and Pseudo-Sherwood treat them in a way which is uniform with their treatment of the original insoluble.

According to Gödel, paradoxical sentences lack meaning. We can say that according to the Ockham-Burley-Pseudo-Sherwood resolution, insolubles lack *expressive completeness*. When Socrates says only 'Socrates says a falsehood' or 'Socrates does not say a truth', he does not thereby succeed in evaluating his utterance.<sup>64</sup> When we reflect on what Socrates says and utter the very same words, we do succeed in evaluating his utterance. Although we use the same words as Socrates, with the same linguistic meaning, Socrates's 'insoluble' utterance is expressively incomplete compared with ours. From our context of utterance, we can evaluate Socrates' utterance; from his context of utterance, he cannot.<sup>65</sup>

In recent years, Charles Parsons and Tyler Burge have proposed indexical solutions to the Liar.<sup>66</sup> Parsons offers an informal account, which, like Burley's, does not limit contextual variation in Liar arguments to the semantical predicates. Such indexicality can arise for 'quantifiers', 'say', 'mean', and other expressions that involve indirect speech'.<sup>67</sup> Parsons writes:

... it seems clear that different occasions of use of a word such as 'say' can presuppose different schemes of interpretation. Moreover, it seems clear that the universe for the quantifiers does not have to be taken as constant for an entire language and even throughout a single discourse, so that this is one dimension with respect to which the schemes of interpretation presupposed in uses of words such as 'say' can differ.<sup>68</sup>

Burge offers a formal theory which is in broad agreement with Parsons' informal account. Burge's formal theory appeals to the indexicality of the truth predicate only, though his indexical view extends to other semantical and epistemic notions.<sup>69</sup> He writes:

In natural language there is a single indexical predicate. We represent this

64 We should note that expressive incompleteness need not be the result of a failure of self-reference; see sub-section 3.1.

65 Herzberger develops a thesis of expressive incompleteness, though not along present lines, in 'Paradoxes of grounding in semantics', *Journal of philosophy*, 67 (1970), 145-167.

66 Parsons, 'The liar paradox', *Journal of philosophical logic*, 3 (1974), 381-412; reprinted with a postscript in R. L. Martin (ed.) *Recent essays on truth and the liar paradox* (Oxford, 1984), 9-45; Burge, 'Semantical Paradox', *Journal of philosophy*, (1979), 169-198 also reprinted with a postscript in the Martin anthology, 83-117, and 'The liar paradox: tangles and chains', *Philosophical studies*, (1982): 352-366.

67 Martin, *ibid.*, 38.

68 *Ibid.*, 34.

69 Burge argues that the terms 'necessary', 'believes', 'knows' and all propositional attitude predicates, are indexical; see his 'Epistemic paradox', *Journal of philosophy*, (1984), 5-29.

predicate by the schematic predicate expression 'true'. This expression may in particular contexts be filled out by any of an unlimited number of numerical subscripts. Any one of the resulting predicates (formally, there are infinitely many) may represent a particular occurrence of 'true' in a context in which its application is fixed. Thus numerals substituted for 'i' mark not new predicate constants, but contextual applications of the indexical 'true'.<sup>70</sup>

Despite the obvious similarities, there are significant differences between these medieval and modern accounts. Unlike Ockham, Pseudo-Sherwood and Burley, Parsons and Burge offer a *hierarchical* account. Parsons suggests that there are more and less comprehensive interpretations of uses of natural language, and '[a] less comprehensive interpretation can be appealed to in a discourse for which a discourse using the more comprehensive interpretation is a metadiscourse'.<sup>71</sup> The indices attached to Burge's univocal, indexical truth predicate correspond to levels which are broadly Tarskian in character.<sup>72</sup> Parsons and Burge admit truth value gaps in a restricted sense,<sup>73</sup> and thus a hierarchy is generated, on pain of the Strengthened Liar. As we have seen, a hierarchy is not to be forced on the bivalent medieval account, at least not from this direction.

A second major difference is that, according to Parsons and Burge, *no* occurrence of the truth predicate is unrestricted.<sup>74</sup> In contrast, according to the Ockham-Burley-Pseudo-Sherwood line there are restrictions on some, but not all, occurrences of semantical predicates; so there is nothing problematic about non-paradoxical, 'global' sentences like 'All sentences are either true or not true', or 'God is omniscient'.<sup>75</sup> Further, as we have seen, the evaluations which cannot be expressed by those sentences in which the semantical predicate is restricted *can* be expressed by other sentences in which the restrictions do not apply: expressive incompleteness is not *absolute*. There is, then, no reason to suppose that the Ockham-Burley-Pseudo-Sherwood resolution is in conflict with the intuition that natural language is *universal*, in Tarski's sense.<sup>76</sup>

Of course, the work of Ockham, Burley and Pseudo-Sherwood on the insolubles does not constitute a fully developed theory of truth; but it is highly suggestive. What their work suggests is an indexical, non-hierarchical and semantically classical

70 Martin (footnote 66), 107.

71 *Ibid.*, 37-38.

72 See, for example, Burge's 'The liar paradox' (footnote 66), 357.

73 See, for example, Burge and Parsons in Martin (footnote 66), 96 and 40 respectively.

74 See Burge and Parsons in Martin (footnote 66), 107 and 28, fn. 13 respectively.

75 To deal with such global sentences, Burge distinguishes between *schematic* and *indexical* occurrences of 'true'. These global sentences are taken to be *schematic* generalizations (see Martin (footnote 66), 107-108, 116). The Ockham-Burley-Pseudo-Sherwood line does not seem to require any such distinction.

76 The intuition that natural language is universal is expressed by Tarski this way: 'A characteristic feature of colloquial language (in contrast to various scientific languages) is its universality. It would not be in the spirit of this language if in some other language a word occurred which could not be translated into it; it could be claimed that "if we can speak meaningfully about anything at all, we can also speak about it in colloquial language"' (Tarski (footnote 54), 164).

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solution to the Liar, which treats our uses of semantical predicates as in need of only minimal corrections. Ockham, Burley and Pseudo-Sherwood offer us something "new", and it warrants further investigation.

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