ON AN ARGUMENT AGAINST OMNISCIENCE

In this paper, I examine a recent argument against omniscience. Patrick Grim has offered an intriguing Cantorian argument for the claim that omniscience is impossible.[1] There are two stages of the argument. The first stage of the argument purports to show that there is no set of all truths. It goes like this:

> There is no set of all truths.
>
> For suppose there were a set T of all truths, and consider all subsets of T, elements of the power set $\mathcal{P}$T.
>
> To each element of this power set will correspond a truth. To each set of the power set, for example, a particular truth T_1 either will or will not belong as a member. In either case we will have a truth: that T_1 is a member of that set, or that it is not. There will then be at least as many truths as there are elements of the power set $\mathcal{P}$T. But by Cantor's power set theorem the power set of any set will be larger than the original. There will then be more truths than there are members of T, and for any set of truths T there will be some truth left out.
>
> There can be no set of all truths.[2]

Now comes the second stage, which concludes that omniscience is impossible:

> Were there an omniscient being, what that being would know would constitute a set of all truths. But there can be no set of all truths, and so can be no omniscient being.[3]

I shall argue that neither stage of Grim's argument succeeds.

I. The bearers of truth

Notice an assumption of the first stage of the argument: "To each element of the power set of T there corresponds a truth". The justification for this assumption is that given any member of the power set of T, there is a truth about that member (for example, that the truth T1 belongs to that member, or that it doesn't). Behind the assumption lies a more general one:

(A) To each set, there corresponds a truth,

justified as a consequence of

(A′) Given any set, there is a truth about that set.

And since sets are just one kind of object, we can generalize the assumption further, to

(B) To each object, there corresponds a truth,

justified as a consequence of

(B′) Given any object, there is a truth about that object.

Should we accept (A) and (A′), and (B) and (B′)?

The question prompts another: what kind of entities are truths? Grim does not consider the matter, so let us survey some principal candidates for the bearers of truth. On one standard view, the truth-bearers are sentences; more precisely, sentence-types in a language.[4] Call a language finite if it has a finite vocabulary and sentences composed of finitely long strings. There may be denumerably many sentences in a finite language; still, there are at most denumerably many truth-bearers. If the truths of this language form a set, this set is at most denumerable. Consider the power set of the set of truth-bearers. By Cantor's theorem, there are more members of this set than there are truth-bearers, and so more members of this power set than there are truths. Here, (A) (and so (A′), (B) and (B′)) are plainly false.

Perhaps, then, we should consider infinitary languages, languages with either an infinitely large vocabulary, or with infinitely long strings, or both. (Here we are clearly moving beyond natural languages.) Take such an infinitary language. As long as there is a cardinal number which measures the size of the vocabulary, and ordinal numbers which measure the length of strings, the sentences of such an infinitary language will form a set. By Cantor's theorem, its power set is larger. And since there are at most as many truths as truth-bearers, there will not be

a truth of the language for every member of the power set, and (A) (and so (A′)) are false. If instead we assume that the sentences of the infinitary language do not form a set, then indeed there will be no set of truths of the language, but this is to assume the result that Grim is out to establish.

What if we suppose that for every cardinal number, there is a language whose set of sentences is of that cardinality? It might seem that under this assumption, (A) is true. Given the set of truths of some language, its power set will outrun the sentences of the language. But there will always be another language which supplies as many truth bearers as there are members of this power set.

Clearly this is a very strong assumption. And now we need no special argument about truths to show that there is no set of all truths. Set-theoretical considerations alone suffice. Simply take a truth from each language; then, by Burali-Forti's theorem and the Axiom of Replacement, there is no set of even these truths, since there is no set of all cardinals.

What is worse, the present assumption together with the assumption (A) generate a paradox. According to (A), there is a truth for each set. It would be better not to take this as saying that there is a truth-in-L for each set, for some language L. For if we suppose that (A) is relativized to the language L, then (A) is false. There is not a truth-in-L for every set. Simply consider the power set of the set of truth-in-L: there will not be a truth-in-L for every member of this power set. Rather, we should understand (A) as:

(1) For any set, there's a corresponding truth in some language.

And we also have this further assumption:

(2) There is a language of every cardinality.

Let us now ask: in what language is the first stage of Grim's argument expressed? Call this language L. (As Grim presents the argument, L appears to be English). In order to express (1), L must have the resources to express the concept true in some language. In order to express (2), L must have the resources to quantify over all cardinals. So, L has the resources to express the concept is true in some language of some cardinality. But this concept generates a version of the Liar paradox. We may construct in L a self-referential sentence which says of itself that it is not true in any language of any cardinality. Assuming that L is a language of some cardinality, a contradiction is forthcoming.

Taking L to be English, the Liar sentence is:

(X) This sentence is not true in any language of any cardinality.

Suppose (X) is a true sentence of English. Then (X) is not true in any language of any cardinality. On the assumption that English is a language of some cardinality, (X) is not a true sentence of English. This is a contradiction. Now suppose (X) is not a true sentence of English. Given that (X) is a sentence of English, it follows that (X) is not true in any language of any cardinality. So what (X) says is the case. So (X) is true; that is, (X) is a true sentence of English. Either way, we obtain a contradiction.

Without a way out of the Liar, the assumption that there is a language of every cardinality will not help. Of course, it could be said that the language of the argument 'stands above' the languages of each cardinality, and is not itself one of them. But this is to assume from the outset a language whose truth-bearers do not form a set, and this begs the question.

Thus far we have assumed that no sentence-type of a language is associated with more than one truth: we have assumed that, if the truths do form a set, the cardinality of sentences is

greater than or equal to the cardinality of truths. We might resist this assumption for at least two reasons: sentences may contain ambiguous expressions, and they may contain context-sensitive expressions. But we can accommodate both ambiguity and context-sensitivity by taking the truth-bearers to be disambiguated sentence-types in a context. Still there will be a set of all truth-bearers, unless we assume further that there are terms ambiguous in as many ways as there are sets, or that there is no set of all contexts. Either assumption would trivialize Grim's argument.

With the indexical term 'here' in mind, we may be led to conclude that there are at least as many truths as there are points in space; thinking of 'now', we may conclude that there are at least as many truths as there points in time; and thinking of the combination 'here now', we might conclude that there are at least as many truths as there are points in space-time. But though we may be led to think that there are uncountably many truths, still this falls short of establishing that there is a truth for every set. The familiar counterargument still works. The set of truth-bearers will be a set of ordered pairs of disambiguated sentence types and contexts, and the power set of this set will have more members than there are truths.

II. Russellian propositions and paradoxes

The foregoing suggests that if truths are linguistic entities, (A) (and so (A′), (B) and (B′)) are false. And Grim's presentation encourages the idea that truths are linguistic items. Given a subset of truths, and a truth T_1, Grim says there is a truth: that T_1 is a member of that subset, or that it isn't. It is natural to suppose that the truth-bearer here is the English sentence "T_1 is a member of that subset" or its negation. And in another presentation of the argument, Grim gives

the following as examples of such truths: "$T_1 \notin \varnothing$", "$T_1 \in \{T_1\}$", "$T_1 \notin \{T_2\}$", ...[5] The construction of a distinct truth for each member of the power set appears to be a linguistic enterprise: we construct appropriate true sentences.

Still, perhaps we can save (A) by breaking the connection between language and truth. Perhaps the truth bearers are non-linguistic propositions. Many different accounts of propositions have been offered, and many of these are perfectly compatible with there being a set of true propositions.[6] But there is a view of propositions according to which, given any set, there is a (true) proposition about that set. Consider Russell's early view of propositions.[7] According to this view, propositions are language-independent structured entities, and objects are constituents of propositions. In correspondence with Frege, Russell insisted that Mont Blanc, "despite all its snowfields",[8] is a part of the proposition expressed by "Mont Blanc is more than 4000 metres high". Now it seems to follow that there cannot be more objects than propositions. Russell argued the point this way:

> ... the number of propositions is just as great as that of all objects absolutely, since every object is identical to itself, and 'x is identical with x' has a one-one relation to x.[9]

To each object x, there corresponds a proposition about x expressed by 'x is identical to x'. Since each of these propositions is true, we obtain (B′), and so (B), (A′) and (A).[10]

But Russell was painfully aware of a problem with the claim that to each set there corresponds a proposition: the claim is implicated in paradox.[11] Given a set S, let us follow Russell and call the proposition expressed by 'Every member of S is true' the <u>logical product</u> of S, and each member of S a <u>factor</u> of this logical product. We just argued that there are just as many propositions as sets. But it seems that there is also a Cantorian proof that there are <u>not</u> as many propositions as sets. Assume, with Russell, the following one-one correlation: each

proposition that is not a logical product is associated with its unit set, the logical product of all propositions is associated with the empty set, and every other logical product is associated with the set of its factors. Russell continues:

> Then the range [i.e. set] w which, by the general principle of Cantor's proof, is not correlated with any proposition, is the range of propositions which are logical products, but are not factors of themselves.[12]

For suppose, towards a contradiction, that there is a proposition correlated with w. Then this correlated proposition is the logical product of w. Is this logical product a factor of itself? If it is, then it is a logical product that is not a factor of itself. If it is not, then it is a logical product that is not a factor of itself, and so is a factor of itself. Either way, we obtain a contradiction. So there is no proposition correlated with the set w, and we have a Cantorian proof that there are more sets than propositions. In particular, there are more sets than true propositions. But this contradicts (A). And we have a paradox.[13]

We might already have suspected that (A) (and (A′)) are associated with paradox. What Grim presents as an argument to the conclusion that there is no set of all truths, others may regard as a paradox - assume that there is a set of all truths, and assume that for every set there corresponds a truth, and a contradiction is generated by a Cantorian argument. Those who find these assumptions natural and intuitive will be unimpressed by Grim's conversion of the paradox to a reductio proof that there is no set of all truths - they will want independent motivation. But the manoeuvre won't work for Russell's paradox about propositions anyway. This is a different paradox: there is no assumption that there is a set of all truths. Russell shows us that (A) is still more deeply involved in paradox.

Russell's paradox about propositions is structurally similar to Cantor's paradox about

sets.[14] We can develop a second closely related paradox about propositions that is structurally similar to Russell's paradox about sets.[15] Some propositions are about sets of propositions (for example, the proposition expressed by 'The set of propositions expressed by Socrates contains profundities'.) And a proposition may be a member of the set of propositions it is about (for an example, suppose that Socrates uttered the last-quoted sentence). Consider the set m of propositions that do not belong to the set of propositions they are about. According to (A′), there is a true proposition about m. But consider a true proposition that is about m. Is this proposition itself a member of m? If it is, then the proposition does belong to the set that it is about, and so is not a member of m. Suppose, on the other hand, that it is not a member of m. Then the proposition does not belong to the set it is about, and so it is a member of m. Either way we get a contradiction. We are landed in paradox. As with Russell's paradox about propositions, the assumption that there is a set of all true propositions is not implicated; but (A′) is.

There are many ways out of these paradoxes. We might deny that propositions can be about sets that contain them, and thereby ban a form of self-reference and circularity. Or we might deny that there there is a set of true logical products that do not belong to their own ranges, or a set of propositions about sets of propositions to which they do not belong. Or we might admit truth gaps, so that, for example, the sentences "w is a factor of itself" and its negation are without truth value. It would take some further argument, beyond anything Grim supplies, to show that these ways out are compatible with (A) and (A′), and incompatible with there being a set of all true propositions.

And some ways out are clearly incompatible with (A) and (A′). We might simply abandon Russellian propositions as truth-bearers, in favor of one of the candidated discussed in

the previous section, and thereby reject (A) directly, along with (A′). Or we might conclude that there is an essential limitation on Russellian propositions: what the paradoxes show is that certain sets cannot be constituents of these structured entities. When we generated Russell's paradox about propositions, we assumed, in accordance with (A), that there is a proposition correlated with w, the set of logical products that are not factors of themselves. This correlated proposition is the logical product of w, the proposition expressed by "Every member of w is true". But we can deny that there can be propositions about w. Similarly, with regard to our second paradox, we can deny that there can be propositions about the set m, the set of propositions that do not belong to the set of propositions they are about. These are sets that cannot be constituents of propositions. There are essential restrictions on what propositions can be about.[16] In this way, we give up (A′), and the justification for (A).

There is an analogue of this way out at the linguistic level. In response to semantical paradox, some have proposed that there are certain sets that are the extension of no linguistic expression: certain sets are beyond the reach of language, on pain of paradox. That is, natural languages like English are essentially expressively incomplete.[17] The thought is that the paradoxical arguments are expressed in English, and paradox results from the attempt to express what is inexpressible in English.

It is not my purpose here to assess these various ways out.[18] My point is that Grim's argument relies on (A) and (A′), and these claims are implicated in semantical paradox. So we are owed a principled way out. And there is, I think, a general lesson here: we cannot establish that there is no set of all truths independently of a solution to the Liar.

III. Omniscience without a set of all truths

Recall the crucial claim of the second stage of Grim's argument: "Were there an omniscient being, what that being would know would constitute a set of all truths." Should we accept this claim?

> Cantor found it necessary to distinguish "two kinds of multiplicities":
>
> For a multiplicity can be such that the assumption that all of its elements "are together" leads to a contradiction, so that it is impossible to conceive of the multiplicity as a unity, as "one finished thing". Such multiplicities I call absolutely infinite or inconsistent multiplicities.
>
> As we can readily see, the "totality of everything thinkable", for example, is such a multiplicity...
>
> If on the other hand the totality of the elements of a multiplicity can be thought of without contradiction as "being together", so that they can be gathered together into "one thing", I call it a consistent multiplicity or a "set".[19]

The multiplicity of all sets would, in Cantor's terminology, be an inconsistent multiplicity: all the sets cannot be thought of as a finished thing, they cannot be gathered together into one thing. This is not to say that they do not all exist. But they do not form a set. Cantor's notion of multiplicity is a notion more comprehensive than that of set. All the sets form a kind of multiplicity that is not a set. Now we might say the same thing about all the truths: they too form a multiplicity that is not a set.

It is reasonable to ask how an inconsistent multiplicity could be given to us, if not as a set, if not as a single thing. As Charles Parsons has suggested, a plausible answer is this: such inconsistent multiplicities are given to us through predication.[20] We can understand the predicate 'is a set', and grasp the concept of a set, in such a way that it is not part of our understanding that the sets together form a single object, or, more strongly, that it is part of our understand-

ing that they do not together form a single object. Parsons goes on to remark: "We might abstract from language and speak with Kant of knowledge through concepts ... "[21] It is sometimes said that Cantor's inconsistent multiplicity is a precursor of the more recent notion of proper class, a kind of collection "too big" to be a set. One way to put the present suggestion is that we take proper classes to be at bottom intensions, rather than any kind of completed collection.[22]

Consider now all the truths. How could God know all the truths if they do not form a set (or any kind of completed totality)? On the present suggestion, God knows all the truths through the extensionless concept under which they fall. God could not grasp all the truths as a single set; there is no such set for God to have knowledge of. We may suppose that God can survey each and every thing that there is, and know whether or not it falls under the concept of truth. The assumption that all the objects that fall under the concept of truth do not together form a single set is no threat to their joint but several existence, or to God's recognition that each falls under the concept. In short, it is no threat to God's omniscience.

It might be denied that there is a concept that applies to all and only the truths. This is another way out of our paradoxes of propositions, where such a concept is assumed. As always, we would need further argument to convince us to take this way rather than the others we have mentioned. We would also need an account of how we are to understand (A), (A′), (B) and (B′), given that the concept of truth is a part of their meaning.

The rejection of a global concept of truth can be further articulated via the notion of a hierarchy. Russell declared that "all propositions" is a "meaningless phrase"[23], and proposed a hierarchy of orders of propositions:

> We may define first-order propositions as those referring to no totality of propositions; second-order propositions, as those referring to totalities of the first order; and so on ad infinitum.[24]

So the meaningless phrase "all propositions" is replaced by a series of meaningful phrases: "all propositions of the first order", "all propositions of the second order", and so on. The global concept of true proposition is rejected in favor of a series of restricted concepts: true first-order proposition, true second-order proposition, and so on.[25] In this way, then, the propositions form a hierarchy, and any of these restricted concepts apply only to the propositions of a single level of the hierarchy.[26]

A related ingredient of the hierarchical approach is that there is no quantification over all propositions. This too constitutes a response to our paradoxes, since both involve unrestricted quantification over propositions. Again, further justification is called for; for one thing, it is not easy to give up a basic principle like 'All propositions are true or not true'. Notice that justification is not provided by the denial that there is a set of all truths. Quantification over all truths does not require a completed totality of truths. D. A. Martin has remarked:

> I don't see why one must believe in infinite sets to understand an assertion like $(\forall x)(\exists y)(\forall z)(\exists w)R(x,y,z,w)$ where R is, say, a molecular formula in the usual number theory. The assertion - and the quantifiers -- do not directly mention any infinite totality. I do not see that the assertion presupposes the existence of any infinite totality. It presupposes only certain finite objects, i.e. each of the natural numbers. Similarly, I do not see how the intelligibility of any particular assertion in the language of formal set theory depends upon the assertion of a completed totality, the class of all sets.[27]

We can say the same thing about quantification over all truths. We do not quantify over a set of truths, only over the truths themselves.[28]

Independent motivation aside, the hierarchical response has difficulties of its own. On the present proposal, the propositions do not form a set, but rather a hierarchy, just as in ZF set

theory there is a hierarchy of all sets, but no set of all sets. One difficulty is that the hierarchy of propositions itself might be regarded as a collection of all the propositions. Why don't the members of the hierarchy form a collection? Why not say that what an omniscient being knows constitutes a hierarchy? The worry is that we've simply rejected one kind of collection (a set) in favor of another. A second difficulty is this: anyone who appeal to a hierarchy cannot admit truths about the hierarchy, for these will be truths involving all propositions. This is not easy to accept. Given the hierarchy, there certainly seem to be truths about it (for example, that its first level contains propositions not about propositions). Indeed, if there are no such truths, we may ask how the hierarchical account is to be stated. Still, let's accept there are no truths about the hierarchy. Then we have an object about which there is no truth, and so (B′) is false, and we lose the justification for (B).

Suppose that, despite these difficulties, we take the hierarchical way out. Suppose that there is no set, or any completed totality, of propositions, and no quantification over all propositions. What are the consequences for omniscience? I see no threat to omniscience. The propositions are there, arranged in a hierarchy, and they fall under God's purview. Of course, the hierarchy is open-ended: at any level α, there are (true) propositions that have not yet appeared at level α or any lower level. But take any true proposition. It belongs to some level σ. Constrained by our finiteness, we cannot run through even denumerably many levels. However, God is presumably under no such constraint. God can run through the ordinals to σ. And however many propositions there are at that level, God can run through them too. Our arbitrary true proposition is accessible to God.

To sum up: even if there is no set of all truths, this does not by itself establish the

impossibility of omniscience. There are ways to reject a set of all truths and accept omniscience. We might admit an extensionless concept of truth, and we might admit a hierarchy of propositions. In the absence of arguments to show that God cannot grasp the concept or survey the hierarchy, the possibility of omniscience remains.

Footnotes

1. The argument appears in two papers of Grim's: "Logic and Limits of Knowledge and Truth", Noûs 1988, p. 356; and "There is no set of all truths", Analysis 1984, pp. 206-207.

2. "Logic and Limits of Knowledge and Truth", p. 356.

3. ibid.

4. A sentence type that is true in one language may not be true in another. Tarski writes:
> We shall also have to specify the language whose sentences we are concerned with; this is necessary if only for the reason that a string of sounds or signs, which is a true or a false sentence but at any rate a meaningful sentence in one language, may be a meaningless expression in another (Alfred Tarski, "Truth and Proof", Scientific American, 1969, p. 86).

5. "There is no set of all truths", p. 207.

6. For some, propositions are tied to sentences. At one time, Russell suggested that a proposition is "the class of all sentences having the same significance as a given sentence" (An Inquiry into Meaning and Truth, p. 502). Propositions are treated as equivalence classes of sentences, and if the sentences form a set, so do these equivalence classes. Quine ties propositions even more tightly to sentences. Quine writes: "I find no good reason not to regard every proposition as nameable by applying brackets to one or another eternal sentence" (W.V.Quine, Word and Object, M.I.T. Press 1960, p. 194). Quine's approach prompts a question: there are eternal sentences that never get uttered, so what of the corresponding proposition? Here is Quine's answer: "We can take each linguistic form as the sequence, in a mathematical sense, of its successive characters or phonemes. A sequence $a_1, a_2,\ldots,a_n$ can be explained as the class of the n pairs $<a_1,1>$, $<a_2,2>$, ..., $<a_n,n>$. We can still take each component character a_i as a class of utterance events, there being no risk of non-utterance." Nothing in this takes us out of the realm of sethood. On both Russell's and Quine's views here, the set of propositions is of the same cardinality as the set of sentences.

Sometimes the debate is couched in terms of whether sentences, statements or propositions are the bearers of truth. Propositions have been preferred because unlike sentences, they do not shift in truth value, and unlike statements, they never fail to have a truth value. Barwise and Etchemendy, for example, reject sentences as truth bearers because different claims may be made by different uses of a sentence (see Jon Barwise and John Etchemendy, The Liar, Oxford University Press, 1987, p. 10), and they reject statements as truth bearers in part because statements, unlike propositions, may fail to have a truth value, as when there is a failure of presupposition (op. cit., p. 11). Barwise and Etchemendy also reject prefer propositions to be truth-bearers on the grounds that statements are too fine-grained. For example, uses of the indexicals 'I' and 'you', allow you and me to express the same truth (that I am tired) using different statements. For Barwise and Etchemendy, "propositions... are the claims made by statements whose presuppositions are fulfilled" (op. cit., p. 12). On this account, there are no

more propositions than there are statements, and so no more propositions than there are "certain sorts of datable events" (op. cit., p. 11). We can only deny that there is a set of propositions if we deny that there is a set of those events "where a speaker asserts or attempts to assert something using a declarative sentence" (op. cit., p. 11). The denial is surely implausible.

7. See Russell, The Principles of Mathematics, Cambridge University Press 1903.

8. See Gottlob Frege, Philosophical and Mathematical Correspondence, edited by H. Hermes et al. (Chicago: University Press 1979), p. 169.

9. The Principles of Mathematics, p. 526, section 498.

10. For Frege, the truth-bearers are thoughts, the senses of sentences. Thoughts do not belong to the realm of reference, but to the realm of sense. Mont Blanc is not a constituent of the thought expressed by "Mont Blanc is more than 4000 metres high"; for Frege, individuals from the realm of reference could never be constituents of thoughts. What is a constituent of the proposition is the sense of the name 'Mont Blanc'. Following Church, let us call the senses of names individual concepts. For Frege, senses are objective and unchanging; they exist independently of us, and of language. So how many senses there are is a matter independent of us and of language.

Now, are there (at least) as many individual concepts as individuals? The answer to this question is independent of limitations of language or thought. It is an ontological question about the objects of the realm of reference and the realm of sense. Since I do not know how Frege would characterize the relation between an individual and an individual concept independently of language and thinkers, I am unsure how to answer the question. But here are two considerations which may suggest that there are no more individuals than individual concepts. First, there are individual concepts for non-existent individuals. And second, on the possible worlds approach, individual concepts are functions from possible worlds to individuals.

If indeed there are at least as many individual concepts as individuals, and if sets are admitted as individuals, then there is no set of individual concepts, since there is no set of all sets. And then, if there is a distinct true thought corresponding to each individual concept, then there is no set of true thoughts: that is, no set of truths, on the Fregean view of truth-bearers.

11. At the time of the writing of The Principles of Mathematics, Russell could see no way out. On p. 368, in section 349, Russell suggests one way out, and rejects it. "In this unsatisfactory state", Russell says, "I reluctantly leave the problem to the ingenuity of the reader".

12. The Principles of Mathematics, p. 527.

13. I have simplified Russell's presentation somewhat. Instead of the term 'set', Russell uses the term 'range'. By this term, Russell means the general notion of a set or class. Russell writes: "It will be convenient to speak of the classes only where we have classes of individuals, of classes of classes only where we have classes of classes of individuals, and so on. For the general notion, I shall use the word range" (The Principles of Mathematics, p. 136). Another complication is that Russell is not univocal in his characterization of logical product. Suppose we

have a class n of propositions. In the above presentation of the paradox, the logical product of n is taken to be the proposition expresed by "Every member of n is true". Russell also characterizes the logical product of n as the conjunction of the members of n (see The Principles of Mathematics, p. 16). However, it is clear that in his presentation of the paradox, Russell is working with the former characterization, as we have.

14. In outline, Cantor's paradox runs as follows: we suppose there is a set U of all sets. Now consider the power set $\mathcal{P}$U of U. There cannot be more sets in $\mathcal{P}$U than in U, since U is the set of all sets. But the familiar Cantorian argument shows that there are more sets in $\mathcal{P}$U than in U, and we have a contradiction. Compare Russell's paradox about propositions, where we claim that there cannot be more sets than propositions, and then, by a Cantorian argument, show that there are.

15. These pairs of paradoxical arguments all share a common structure: they are all diagonal arguments. For a general characterization of the diagonal argument, see my "The Diagonal Argument and the Liar", Journal of Philosophical Logic, 1990.

16. In rejecting Russell's hierarchical response to the paradoxes, Gödel says: "It might even turn out that it is possible to assume every concept to be significant everywhere except for certain 'singular points' or limiting points', so that the paradoxes would appear as something analogous to dividing by zero." One might try to adapt Gödel's remarks to the present setting, and claim that propositions can be about any object except for certain singularities, like the sets m and w.

17. Hans Herzberger has suggested such a way out of semantical paradox, in "Paradoxes of Grounding in Semantics", Journal of Philosophy, 1970, and "New Paradoxes for Old", Proceedings of the Aristotelian Society, 1981. For example, Herzberger argues that, relative to a simple view of concepts, there is no term of English whose extension is exactly those terms of English not true of themselves.

18. I have critically examined a number of current ways out of the Liar in "The Diagonal Argument and the Liar", loc. cit.

19. Georg Cantor, Letter to Dedekind (1899), translated in Jean van Heijenoort, From Frege to Gödel, Harvard University Press 1967, p. 114.

20. See Charles Parsons, "What is the iterative conception of set?", in Readings in the Philosophy of Mathematics, 2nd edition, edited by Paul Benacerraf and Hilary Putnam, Cambridge University Press 1983, p. 516.

21. ibid.

22. See for example, Parsons, op. cit., p. 518. Grim does consider some treatments of proper classes (the ones found in the alternative set theories of Quine, and von Neumann and Bernays), but does not consider the intensional treatment. On another line that Grim does not consider, proper classes are again associated with predication, but are taken to be the extensions of

predicates, rather than their intensions. Classes are treated according to the predicative conception, while sets are treated according to the iterative conception. And paradoxes about proper classes are avoided by allowing truth gaps in the membership relation between classes. See D. A. Martin, "Sets versus Classes" (circulated mimeograph) and Penelope Maddy, "Proper Classes", Journal of Symbolic Logic, 1984.

23. Russell, "Mathematical logic as based on the theory of types", in van Heijenoort (ed.), p. 154.

24. Russell, My Philosophical Development, Simon and Schuster 1959, p. 82. See also "Mathematical logic as based on the theory of types", loc. cit., p. 164.

25. Compare Tarski's stratification of the truth predicate into infinitely many predicates, each belonging to a language in a hierarchy of object languages and metalanguages. Russell remarks that his hierarchical theory and the doctrine of a hierarchy of languages belong to "the same order of ideas" (Russell, Logic and Knowledge, Capricorn Books 1971, p. 371). According to Church, "Russell's resolution of the semantical antinomies is not a different one than Tarski's but is a special case of it" (Alonzo Church, "Comparison of Russell's resolution of the Semantical Antinomies with that of Tarski", Journal of Symbolic Logic, vol. 41, Number 4, Dec 1976, pp. 747-60). The Tarskian approach is perhaps the closest we have to an orthodoxy concerning semantical paradox.

26. In written and oral discussion, Grim has also endorsed a hierarchical response to semantical (and set-theoretical) paradoxes.

27. D. A. Martin, "Sets versus Classes", circulated mimeograph, pp. 2-3.

28. Lewis makes similar remarks with regard to the sentence, "There are all the non-self-members, and they do not comprise any sort of set or class". Lewis finds it plausible to say of this quantification that "(I) am not quantifying in any ordinary way over any set or class of ... non-self-members; rather I am quantifying over nothing but ... non-self-members themselves, however I am quantifying over them in an irreducibly plural way" (The Plurality of Worlds, Basil Blackwell 1986, p. 51, fn. 37.)